International Journal of Theoretical Physics, Vol. 46, No. 5, May 2007 (© 2007)
DOI: 10.1007/s10773-006-9275-0

Quasi-Particle Theory of Alfven Soliton Interaction in Plasmas

Yan Xiao[1] and Anjan Biswas[2]

Received August 25, 2006; accepted September 26, 2006
Published Online: February 22 2007

The collision of solitons due to Alfven waves in plasmas is studied in this paper by the aid of quasi-particle theory. The suppression of the interaction of solitons, in presence of the perturbation terms, is acheived by means of this theory. The perturbation terms that are considered in this paper are nonlinear damping, finite conductivity and Landau damping. The numerical simulations support the theory that was developed.

KEY WORDS: solitons; quasi-particle theory; Alfven waves; Landau damping.
PACS Codes: 02.30.Ik, 02.30.Jr, 52.35.Sb.

1. INTRODUCTION

The study of solitons in the context of Plasma Physics is governed by the derivative nonlinear Schrödinger's equation (DNLSE) (Ablowitz and Clarkson, 1991; Biswas, 2005; Fla and Mjolhus, 1989; Mamun, 1999; Mjolhus and Wyller, 1988). The dimensionless form of DNLSE is given by

$$q_t + iq_{xx} + (|q|^2 q)_x = 0 \tag{1}$$

Equation (1) is integrable (Ablowitz and Clarkson, 1991; Kaup and Newell, 1978; Wyller and Mjolhus, 1984) by Inverse Scattering Transform (IST) since it passes the Painleve test of integrability (Ablowitz and Clarkson, 1991).

The DNLSE has historically found applications in many areas of physics, one example being circularly polarized nonlinear Alfveń waves in plasmas. Related models have recently received fresh attention in the context of chiral Luttinger liquids; some of these models can be obtained by a dimensionl reduction of a Chern–Simmons model defined in two dimensions. The DNLSE has some

[1] College of Physics & Electronics Engineering and the State Key Subject of Optics, Shanxi University, Taiyuan-030006, China.
[2] Department of Applied Mathematics and Theoretical Physics, Delaware State University, Dover, DE 19901-2277, USA.

pecularities, for instance, it is not Galilean invariant and it has classical solitons which have an upper bound on the particle number and are chiral (with a particular sign of momentum). In these aspects DNLSE differs from the usual nonlinear Schrödinger's equation although both of them are classically integrable (Basu-Mallick *et al.*, 2003, 2004).

The DNLSE is used for modeling of wave processes in different physical systems such as nonlinear optics, Stokes waves in fluids of finite depth and many others. In nonlinear optics, DNLSE can be derived in a systemetic way by means of reductive perturbation scheme as a model for single mode propagation.

Moreover, DNLSE arises in the study of the collective processes in dusty plasmas, namely plasmas with extremely massive and highly charged dust grains which are ubiquitous in laboratory, space and astrophysical plasma environments such as cometary tails, asteroid zones, planetary rings, interstellar medium, earth's environment, just to name a few. It has been shown, both theoretically and experimentally, that the presence of extremely massive and highly charged static dust grains modifies the existing plasma wave spectra. A substantial number of investigations have already been made on linear and nonlinear properties of the electrostatic modes in dusty plasmas with or without external magnetic field (Mamun, 1999).

Recently, there has been rapidly growing interest in the study of different types of new electromagnetic modes in dusty plasmas and a limited number of attempts on these electromagnetic modes have been made by a number of authors. The linear analysis of electromagnetic waves, propagating perpendicular to the external magnetic field, in a multi-species dusty plasma was carried out. The low frequency electromagnetic Alfveń mode, in dusty cometary and planetary plasmas, was also studied (Mamun, 1999).

2. MATHEMATICAL MODEL

The perturbed DNLSE that is going to be studied in this paper for the soliton-soliton interaction (SSI) is

$$q_t + iq_{xx} + (|q|^2 q)_x = \epsilon R[q, q^*] \tag{2}$$

where

$$R = \delta |q|^{2m} q + \beta q_{xx} + \lambda \frac{\partial}{\partial x} \left\{ q(x,t) \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{|q(s,t)|^2}{s-x} ds \right\} \tag{3}$$

In (2), R represents the perturbation terms while the perturbation parameter ϵ satisfies $0 < \epsilon \ll 1$. Here, in (3), the coefficient of δ represents the nonlinear gain (damping) depending on whether δ is positive or negative. Also, the coefficient of β arises when finite conductivity is included in the reductive perturbation treatment of the Alfveń waves in plasmas (Boling and Yaping, 1995; Mjolhus and Wyller, 1988; Wyller and Mjolhus, 1984). Moreover, the non-local perturbation term with λ accounts for nonlinear Landau damping (Ding and Zhu, 2002; Fla and Mjolhus, 1989; Gazol *et al.*, 1999) where P represents the principal value of the integral. This perturbation term also plays an important role in collisionless plasmas. It is to be noted that the coefficient λ, for cold plasmas, is zero (Wyller and Mjolhus, 1984).

The quasi-particle theory (QPT) of SSI will be investigated and it will be proved by virtue of it that the interaction can be supressed due to the presence of perturbation terms in (2).

The soliton solution of (1) as obtained by IST is given by

$$q(x,t) = \frac{A}{[\kappa + a\ \cosh\{A(x-\bar{x})\}]^{\frac{1}{2}}} e^{i(\kappa x - \omega t + \sigma_0)} \tag{4}$$

where

$$A = 2\sqrt{\kappa^2 + \omega} \tag{5}$$

$$\omega = \frac{A^2}{4} - \kappa^2 \tag{6}$$

$$a = \sqrt{\omega + 2\kappa^2} \tag{7}$$

$$v = \frac{d\bar{x}}{dt} = -2\kappa \tag{8}$$

Here in (4), A represents the soliton amplitude or the inverse width of the soliton, κ is the frequency while ω is the wave number of the soliton. Finally, $\bar{x}$ and σ_0 are the center of the soliton and center of phase of the soliton respectively, so that v gives the velocity of the soliton that is related to the frequency as given in (8).

Also, the 2-soliton solution of the DNLSE (1) takes the asymptotic form (Biswas, 1999)

$$q(x,t) = \sum_{l=1}^{2} \frac{A_t}{[\kappa_l + a_l \cosh\{A_l(x-\bar{x}_l)\}]^{\frac{1}{2}}} e^{i(\kappa_l x - \omega_l t + \sigma_{0_l})} \tag{9}$$

where

$$A_t = 2\sqrt{\kappa_l^2 + \omega_l} \tag{10}$$

$$\omega_l = \frac{A_l^2}{4} - \kappa_l^2 \tag{11}$$

$$a_l = \sqrt{\omega_l + 2\kappa_l^2} \tag{12}$$

In the study of SSI, the initial waveform is taken to be of the form

$$q(x,0)=\frac{\eta_1}{\left[\kappa_1+a_1\cosh\left\{\eta_1\left(x-\frac{x_0}{2}\right)\right\}\right]^{\frac{1}{2}}}e^{i\phi_1}+\frac{\eta_2}{\left[\kappa_2+a_2\cosh\left\{\eta_2\left(x+\frac{x_0}{2}\right)\right\}\right]^{\frac{1}{2}}}e^{i\phi_2} \tag{13}$$

which represents initial 2-soliton like waves. Here, x_0 represents the initial separation of the solitons. It needs to be noted that for $x_0 \to \infty(9)$ represents exact soliton solutions, but for $x_0 \sim O(1)$ it does not represent an exact 2-soliton solution. Corresponding to the input waveform given by (13) the case of in-phase input of solitons with equal amplitudes will be studied. Thus without any loss of generality the choice $\eta_1 = \eta_2 = 1$ and $\phi_1 = \phi_2 = 0$ is considered so that (13) modifies to

$$q(x,0)=\frac{\sqrt{2}}{\cosh^{\frac{1}{2}}\left(x-\frac{x_0}{2}\right)}+\frac{\sqrt{2}}{\cosh^{\frac{1}{2}}\left(x+\frac{x_0}{2}\right)} \tag{14}$$

3. QUASI-PARTICLE THEORY

The QPT dates back to 1981 since the appearance of the paper by Karpman and Solov'ev (2000). The mathematical approach to the SSI using the QPT will be studied in this paper. Here, the solitons are treated as particles. If two waveforms are separated and each of them is close to a soliton they can be written as the linear superposition of two soliton like waveforms (Biswas, 1999) given by

$$q(Z,T)=q_1(Z,T)+q_2(Z,T) \tag{15}$$

with

$$q_l(x,t)=\frac{A_t}{[B_l+\alpha_l\cosh[A_l(x-x_l)]]^{\frac{1}{2}}}e^{-iB_l(x-x_l)+i\delta_l} \tag{16}$$

where $l = 1, 2$ and A_l, B_l, x_l and δ_l are functions of t. It needs to be noted that A_l, B_l do not represent the amplitude, and frequency of the full wave form. However, they approach the amplitude and frequency respectively for large separation namely as $\Delta x = x_1 - x_2 \to \infty$, then $A_l \to \eta_l$ and and $B_l \to \kappa_l$. Since the waveform is assumed to remain in the form of two solitons, the method is called the quasi-particle approach. First, the equations for A_l, B_l, x_1 and δl will be derived using the soliton perturbation theory (SPT) (Biswas, 1999). Substituting (15) into (2) gives

$$\frac{\partial q_l}{\partial t}+i\frac{\partial^2 q_l}{\partial x^2}+\frac{\partial}{\partial x}(|q_l|^2 q_l)=i\epsilon R[q_l,q_l^*]-\frac{\partial}{\partial x}\left(2|q_l|^2 q_{\bar{l}}+q_l^2 q_{\bar{l}}^*\right) \tag{17}$$

where $l = 1, 2$ and $\bar{l} = 3 - l$. Here, the separation

$$(|q|^2q)_x=\left[\left(|q_1|^2q_1+q_1^2q_2^*+2|q_1|^2q_2\right)+\left(|q_2|^2q_2+q_2^2q_1^*+2|q_2|^2q_1\right)\right]_x \tag{18}$$

was used based on the degree of overlapping. By SPT, the evolution equations are

$$\frac{dA_l}{dt} = F_1^{(l)}(A, \Delta x, \Delta\phi) + \epsilon M_l \tag{19}$$

$$\frac{dB_l}{dt} = F_2^{(l)}(A, \Delta x, \Delta\phi) + \epsilon N_l \tag{20}$$

$$\frac{dx_l}{dt} = -B_l - F_3(A, \Delta x, \Delta\phi) + \epsilon Q_l \tag{21}$$

$$\frac{d\delta_l}{dt} = B_l - \frac{A_l^2}{4} + F_4(A, \Delta x, \Delta\phi) + \epsilon P_l \tag{22}$$

where, the functions $F_1^{(l)}$, $F_2^{(l)}$, F_3 and F_4 formulate on using the SPT in (17), with the right side being treated as perturbation terms and

$$M_l = h_l(A_l)\int_{-\infty}^{\infty} \Re\{\hat{R}[q_l, q_l^*]e^{-i\phi_l}\}\frac{d\tau_l}{(B_l + a_l\cosh\tau_l)^{\frac{1}{2}}}d\tau_l \tag{23}$$

$$N_l = h_2(A_l)\int_{-\infty}^{\infty} \Im\{\hat{R}[q_l, q_l^*]e^{-i\phi_l}\}\frac{\sinh\,\tau_l}{(B_l + a_l\cosh\tau_l)^{\frac{1}{2}}}d\tau_l \tag{24}$$

$$Q_l = h_3(A_l)\int_{-\infty}^{\infty} \Re\{\hat{R}[q_l, q_l^*]e^{-i\phi_l}\}\frac{\tau_l}{(B_l + a_l\cosh\tau_l)^{\frac{1}{2}}}d\tau_l \tag{25}$$

$$P_l = h_4(A_l)\int_{-\infty}^{\infty} \Im\{\hat{R}[q_l, q_l^*]e^{-i\phi_l}\}\frac{(1 - a_l\tau_l\sinh\tau_l)}{(B_l + a_l\cosh\tau_l)^{\frac{1}{2}}}d\tau_l \tag{26}$$

Here, the functions $h_j(A_l)$ for $1 \le j \le 4$ are by virtue of (19)–(22) and $\Re$ and $\Im$ stands for the real and imaginary parts respectively. Also, the following notations are used (Mamun, 1999; Mio *et al.*, 1976; Mjolhus and Wyller, 1988; Ozawa, 1996; Ruderman, 2002; Sen and Chowdhury, 1987; Wyller and Mjolhus, 1984)

$$\hat{R}[q_l, q_l^*] = R[q_l, q_l^*] - \frac{\partial}{\partial x}\left(q_l^2 q_{\bar{l}}^* + 2|q_l|^2 q_{\bar{l}}\right) \tag{27}$$

$$\tau_l = A_l(x - x_l) \tag{28}$$

$$\phi_l = B_l(x - x_l) - \delta_l \tag{29}$$

$$\Delta\phi = B\Delta x + \Delta\delta \tag{30}$$

$$\Delta x = x_1 - x_2 \tag{31}$$

$$\Delta\delta = \delta_1 - \delta_2 \tag{32}$$

$$A = \frac{1}{2}(A_1 + A_2) \tag{33}$$

$$B = \frac{1}{2}(B_1 + B_2) \tag{34}$$

$$\Delta A = A_1 - A_2 \tag{35}$$

$$\Delta B = B_1 - B_2 \tag{36}$$

Moreover, it was assumed that

$$|\Delta A| \ll A \tag{37}$$

$$|\Delta B| \ll 1 \tag{38}$$

$$A\Delta x \gg 1 \tag{39}$$

$$|\Delta A|\Delta x \ll 1 \tag{40}$$

From (19) to (22) one can now obtain

$$\frac{dA}{dZ} = \epsilon M \tag{41}$$

$$\frac{dB}{dZ} = \epsilon N \tag{42}$$

$$\frac{d(\Delta A)}{dt} = F_1^{(1)}(A, \Delta x, \Delta\phi) - F_1^{(2)}(A, \Delta x, \Delta\phi) + \epsilon\Delta M \tag{43}$$

$$\frac{d(\Delta B)}{dt} = F_2^{(1)}(A, \Delta x, \Delta\phi) - F_2^{(2)}(A, \Delta x, \Delta\phi) + \epsilon\Delta N \tag{44}$$

$$\frac{d(\Delta\phi)}{dt} = -\Delta B + \epsilon\Delta Q \tag{45}$$

$$\frac{d(\Delta\phi)}{dt} = \frac{1}{2}A\Delta A + \frac{\Delta x}{2}\left(F_2^{(1)} + F_2^{(2)}\right) + \epsilon B\Delta Q + \epsilon\Delta P \tag{46}$$

where

$$M = \frac{1}{2}(M_1 + M_2) \tag{47}$$

$$N = \frac{1}{2}(N_1 + N_2) \tag{48}$$

and ΔM, ΔN, ΔQ and ΔP are the variations of M, N, Q and P which are written as for example

$$\Delta M = \frac{\partial M}{\partial A}\Delta A + \frac{\partial M}{\partial B}\Delta B \tag{49}$$

assuming that they are functions of A and B only, which is, in fact, true for most of the cases of interest, otherwise, the equations for

$$T = \frac{1}{2}(T_1 + T_2) \tag{50}$$

and

$$\phi = \frac{1}{2}(\phi_1 + \phi_2) \tag{51}$$

would have been necessary. In presence of the perturbation terms, as given by, (3), the dynamical system of the soliton parameters, by virtue of soliton perturbation theory, are (Ablowitz and Clarkson, 1991; Biswas, 1999, 2003)

$$\begin{aligned}
\frac{dA}{dt} = &\frac{\epsilon}{E}\left[\frac{\delta A^{2m+2}}{2^m a^{m+1}} F\left(m+1, m+1, m+\frac{3}{2}; \frac{a-B}{2a}\right) B\left(m+1, \frac{1}{2}\right)\right.\\
&-\frac{\beta A^2 B^2}{a} F\left(1, 1, \frac{3}{2}; \frac{a-B}{2a}\right) B\left(1, \frac{1}{2}\right)\\
&-\frac{\beta A^4}{4a^2} F\left(2, 2, \frac{5}{2}; \frac{a-B}{2a}\right) B\left(2, \frac{1}{2}\right)\\
&+\frac{\beta A^4 B}{8a^3} F\left(3, 3, \frac{7}{2}; \frac{a-B}{2a}\right) B\left(3, \frac{1}{2}\right)\\
&\left.+\frac{\beta A^4 B}{4a} F\left(3, 1, \frac{7}{2}; \frac{a-B}{2a}\right) B\left(1, \frac{3}{2}\right)\right]\\
&+\frac{2\epsilon\lambda A^3}{\pi E}\int_{-\infty}^{\infty}\frac{1}{B + a\cosh\tau}\left\{\frac{\partial}{\partial x} P\int_{-\infty}^{\infty}\frac{d\tau_1}{(s-x)(B + a\cosh\tau_1)}\right\} d\tau\\
&+\frac{\epsilon}{4E}(A^2 E + 4B^2 E - 8AB)\\
&\left[\frac{\delta A^{2m+1}}{2^m a^{m+1}} F\left(m+1, m+1, m+\frac{3}{2}; \frac{a-B}{2a}\right) B\left(m+1, \frac{1}{2}\right)\right.\\
&-\frac{\beta A B^2}{a} F\left(1, 1, \frac{3}{2}; \frac{a-B}{2a}\right) B\left(1, \frac{1}{2}\right) - \beta a A^3 F\left(1, 1\frac{5}{2}; \frac{a-B}{2a}\right)\\
&\times B\left(1, \frac{3}{2}\right) + \frac{\lambda A}{\pi}\int_{-\infty}^{\infty}\frac{1}{B + a\cosh\tau}\\
&\times\left\{\frac{\partial}{\partial x} P\int_{-\infty}^{\infty}\frac{d\tau_1}{(s-x)(B + a\cosh\tau_1)}\right\} d\tau - \frac{a\lambda A^2}{2\pi}\\
&\left.\times\int_{-\infty}^{\infty}\frac{\sinh\tau}{(B + a\cosh\tau)^2}\left\{P\int_{-\infty}^{\infty}\frac{d\tau_1}{(s-x)(B + a\cosh\tau_1)}\right\} d\tau\right]
\end{aligned} \tag{52}$$

$$\frac{dB}{dt} = \frac{\epsilon\beta AB}{2a^2E}\left[A^2BF\left(2,2,\frac{5}{2};\frac{a-B}{2a}\right)B\left(2,\frac{1}{2}\right)\right.$$
$$-\,2a\left(A^2+4B^2\right)F\left(1,1,\frac{3}{2};\frac{a-B}{2a}\right)B\left(1,\frac{1}{2}\right)\Bigg]+\frac{\epsilon aAB\lambda}{\pi E}$$
$$\left.\times\int_{-\infty}^{\infty}\frac{\sinh\tau}{(B+a\ \cosh\ \tau)^2}\left\{P\int_{-\infty}^{\infty}\frac{d\tau_1}{(s-x)(B+a\ \cosh\ \tau_1)}\right\}d\tau\right] \tag{53}$$

where $\tau_1 = A(s-\bar{x})$ and E is the energy of the unperturbed soliton, given by

$$E = \int_{-\infty}^{\infty}|q|^2dx = \frac{A}{2a}F\left(1,1,\frac{3}{2};\frac{a-\kappa}{2a}\right)B\left(1,\frac{1}{2}\right) = 4\tan^{-1}\left(\frac{A}{2\kappa}\right) \tag{54}$$

and $F(\alpha,\beta;\gamma;z)$ is the Gauss' hypergeometric function defined as

$$F(\alpha,\beta;\gamma;z) = \frac{\Gamma(\gamma)}{\Gamma(\alpha)\Gamma(\beta)}\sum_{n=0}^{\infty}\frac{\Gamma(\alpha+n)\Gamma(\beta+n)}{\Gamma(\gamma+n)}\frac{z^n}{n!} \tag{55}$$

and $B(l,m)$ is the usual beta function. For $A(t)=1$ and $B(t)=0$, the dynamical system given by (52) and (53) has a stable fixed point provided

$$\beta = \frac{2\lambda\int_{-\infty}^{\infty}\frac{2}{\cosh\tau}\left\{\frac{\partial}{\partial x}\int_{-\infty}^{\infty}\frac{2d\tau_1}{(s-x)\cosh\tau_1}\right\}d\tau}{\pi F\left(2,2,\frac{5}{2};\frac{1}{2}\right)B\left(2,\frac{1}{2}\right)}$$
$$+\frac{2\delta F\left(m+1,m+1,m+\frac{3}{2};\frac{1}{2}\right)B\left(m+1,\frac{1}{2}\right)}{F\left(2,2,\frac{5}{2};\frac{1}{2}\right)B\left(2,\frac{1}{2}\right)} \tag{56}$$

Thus, one can obtain using, (45), (46) and (53)–(55)

$$\frac{d^2(\Delta x)}{dt^2} + \epsilon\beta G\frac{d(\Delta x)}{dt} + F_2^{(1)} - F_2^{(2)} = 0 \tag{57}$$

where, $G > 0$ represents the coefficient of $-\epsilon\beta\Delta B$ in $d(\Delta B)/dt = dB_1/dt - dB_2/dt$ and β is given by (56). Now, Eq. (57) shows that there is a damping in the separation of solitons, thus proving that there will be a suppression of the SSI in presence of the perturbation terms given by (3).

4. NUMERICAL SIMULATIONS

The mathematical set up, given by (52) to (57), will be used to study the supression of SSI by numerical simulation. In Figs. 1–3, the choices $\epsilon = 0.1$, $A_1 = A_2 = 0.8$, $a_1 = a_2 = 0.1$, $B_1 = B_2 = 1.2$ and $x_1 = -x_2 = 8$ were made, so that $\Delta x = 16$. In all the simulations, the solitons' transmission distance was taken to be $t = 10$.

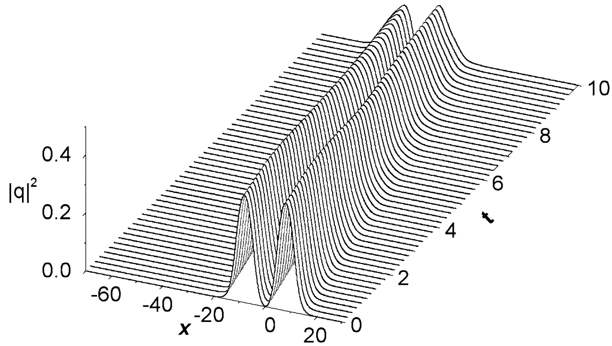

Fig. 1. $m = 0$, $\delta = 1$ and $\beta = 1.57$.

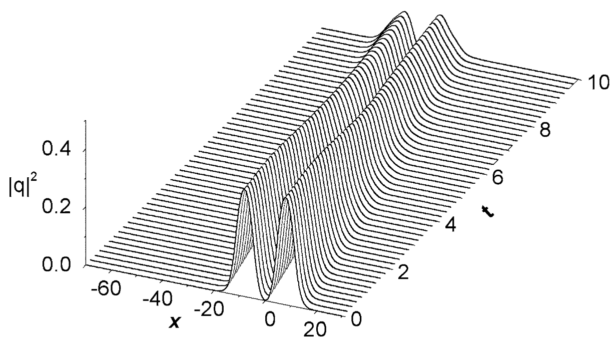

Fig. 2. $m = 1$, $\delta = 1$ and $\beta = 2$.

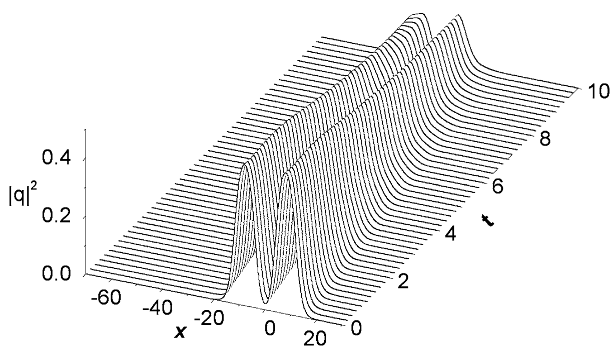

Fig. 3. $m = 2$, $\delta = 1$ and $\beta = 3.14$.

5. CONCLUSIONS

In this paper, the SSI of the Alfvén waves in plasmas is studied. The suppression of the SSI is acheived and this is proved by the quasi-particle theory of SSI. The numerical results also support the theory that was developed in this paper. In future one can take a look into other perturbation terms and their effects on SSI and how it can be suppressed. Such results will be reported in a future publication.

ACKNOWLEDGMENTS

This research for the second author was fully supported by NSF Grant No: HRD-970668 and the support is thankfully appreciated.

REFERENCES

Ablowitz, M. J. and Clarkson, P. A. (1991). *Solitons Nonlinear Evolution Equations and the Inverse Scattering Transform*. (Cambridge University Press, Cambridge).

Biswas, A. (1999). Soliton-soliton interaction in optical fibers. *Journal of Nonlinear Optical Physics and Materials* **8**(4), 483–495.

Biswas, A. (2003). Quasi-stationary non-Kerr law optical solitons. *Optical Fiber Technology* **9**(4), 224–259.

Biswas, A. (2005). Soliton perturbation theory for Alfvén waves in plasmas. *Physics of Plasmas* **12**(2), 022306.

Basu-Mallick, B., Bhattacharyya, T., and Sen, D. (2003). Novel multi-band quantum soliton states for a derivative nonlinear Schrödinger model. *Nuclear Physics B* **675**(3), 516–532.

Basu-Mallick, B., Bhattacharyya, T., and Sen, D. (2004). Bound and anti-bound soliton states for a quantum integrable derivative nonlinear Schrödinger model. *Physics Letters A* **325**(5–6), 375–380.

Boling, G. and Yaping, W. (1995). Orbital stability of solitary waves for the nonlinear derivative Schrödinger equation. *Journal of Differential Equations* **123**, 35–55.

Chen, X. J. and Yang, J. (2002). Direct perturbation theory for solitons of the derivative nonlinear Schrödinger equation and the modified nonlinear Schrödinger equation. *Physical Review E* **65**, 066608.

Ding, Q. and Zhu, Z. (2002). On the gauge equavalent structure of the modified nonlinear Schrödinger equation. *Physics Letters A* **295**, 192–197.

Fla, T. and Mjolhus, E. (1989). Nonlinear landau damping of weakly dispersive circularly polarized MHD waves. *Physica Scripta* **40**, 219–226.

Gazol, A., Passot, T., and Sulem, P. L. (1999). Coupling between nonlinear Alfvén waves and reduced magnetohydrodynamics for compressible fluids. *Physics of Plasmas* **6**(8), 3114–3122.

Gerdjikov, V. S., Doktorov, E. V., and Yang, J. (2001). Adiabatic interaction of Nultrashort solitons: Universality of the complex Toda chain model. *Physical Review E* **64**, 056617.

Hong, L., Beech, R., Osman, F., Xian-Tu, H., Sen-Yue, L., and Hora, H. (2004). Periodic and solitary waves of the cubic-quintic nonlinear Schrödinger equation. *Journal of Plasma Physics* **70**, 415–429.

Karpman, V. I. (2000). Radiation of solitons by a high-order cubic nonlinear Schrödinger equation. *Physical Review E* **62**(4), 5678–5687.

Kaup, D. J. and Newell, A. C. (1978). An exact solution for a derivative nonlinear Schrödinger equation. *Journal of Mathematical Physics* **19**(4), 798–801.

Lee, J. H., Lee, Y. C., and Lin, C. C. (2002). Exact solutions of DNLS and Derivative Reaction-Diffusion Systems. *Journal of Nonlinear Mathematical Physics* **9**(Suppl. 1), 87–97.

Mamun, A. A. (1999). Alfvén solitary structures and their instabilities in a magnetized dusty plasma. *Physica Scripta* **60**, 365–369.

Mio, W., Ogino, T., Minami, K., and Takeda, S. (1976). Modified nonlinear Schrödinger equation for Alfvén waves propagating along the magnetic field in cold plasmas. *Journal of Physical Society of Japan* **41**, 265–271.

Mjolhus, E. and Wyller, J. (1988). Nonlinear Alfvén waves in finite beta-plasma. *Journal of Plasma Physics* **40**(Part 2), 299–318.

Ozawa, T. (1996). On the nonlinear Schrödinger equations of derivative type. *Indiana University Mathematics Journal* **45**, 137–164.

Ruderman, M. S. (2002). Propagation of solitons of the derivative nonlinear Schrödinger equation in a plasma with fluctuating density. *Physics of Plasmas* **9**(7), 2940–2945.

Sen, S. and Chowdhury, A. R. (1987). On a Bethe-ansatz approach to the derivative nonlinear Schrödinger equation. *Journal of Physics A* **20**, L1089–L1084.

Wyller, J. and Mjolhus, E. (1984). A perturbation theory for Alfven waves. *Physica D* **13**, 234–246.

Wyller, J., Fla, T., and Mjolhus, E. (1989). The effect of resonant particles on Alfvén solitons. *Physica D* **39**(2–3), 405–422.

Xiao, Y. (1991). Note on the Darboux transformation for the derivative nonlinear Schrödinger equation. *Journal of Physics A* **24**, 363–366.